and compact, and from this it follows that the set function is countably additive. As a historical note, we remark that the concept of compact measure was introduced by Marczewski (1953).

Arbitrary Sums of I.R.V. We can now prove the main theorem of this chapter.

(1.6) Theorem. Let X be a locally compact topological semi-group and let $\mathcal{J}$ be the Borel sets of X. If $\{m_\alpha : \alpha \varepsilon I\}$ is an arbitrary family of regular probability measures on X, then there is a probability space $(\Omega, \mathcal{E}, \mu)$ and a collection of i.r.vs. $\{\xi_\alpha : \alpha \varepsilon I\}$ such that m_α is the distribution of ξ_α for each α in I and all finite sums of i.r.vs. taken from among this collection are r.vs.

Proof. Let $\Omega = X^I$ and let $\{\xi_\alpha : \alpha \varepsilon I\}$ be the canonical projection maps. Let $\mathcal{B}$ be the class of all sets in Ω of the form

$$\{x : \xi_{\alpha_1 \dots \alpha_n}(x) \varepsilon E\}$$

Where $\xi_{\alpha_1 \dots \alpha_n}$ is the canonical projection $X^I \to X^n$ defined by

$$\xi_{\alpha_1 \dots \alpha_n}(x) = (\xi_{\alpha_1}(x), \dots, \xi_{\alpha_n}(x)),$$

n is any integer and E is any Borel set of X^n. Define $\mathcal{C}$ to be the class of all sets in $\mathcal{B}$ for which E is a compact set. Then $\mathcal{B}$ is a semi-algebra and $\mathcal{C}$ is a compact class. Define the finitely additive set function μ' on $\mathcal{B}$ via:

$$\mu'\{x : \xi_{\alpha_1 \dots \alpha_n}(x) \varepsilon E\} = (\Pi_{j=1}^n m_{\alpha_j})(E).$$

If $\mathcal{C}$ is a collection of ordinals, then lub $\mathcal{C}$ (the <u>least upper bound of $\mathcal{C}$</u>) $=_{df}$ the least ordinal α such that $\forall\beta_{\beta\in\mathcal{C}}[\beta \leq \alpha]$; and lob $\mathcal{C}$ $=_{df}$ the least ordinal α such that $\forall\beta_{\beta\in\mathcal{C}}[\beta < \alpha]$. Thus lub $\{0,1,2\} = 2$, but lob $\{0,1,2\} = 3$.

If λ is a limit ordinal, then a <u>λ-type fundamental sequence</u> is an indexed collection of ordinals $\{\alpha_\gamma : \gamma < \lambda\}$ such that $\beta < \gamma < \lambda$ implies $\alpha_\beta < \alpha_\gamma$. If $\{\alpha_\gamma : \gamma < \lambda\}$ is a fundamental sequence, then $\lim_{\gamma<\lambda} \alpha_\gamma$ (the <u>limit</u> of the sequence) $=_{df}$ lob $\{\alpha_\gamma : \gamma < \lambda\}$.

For any collection B, card (B) is the cardinality of B. Thus card (N) $= \aleph_0$. B is <u>denumerable</u> iff card (B) $= \aleph_0$, and B is <u>countable</u> iff card (B) $\leq \aleph_0$. For every ordinal α, card (α) $=_{df}$ card $\{\beta : \beta < \alpha\}$. If we categorize the ordinals in terms of cardinality, then $\{n: n < \omega\}$ is the class of finite ordinals and $\{\alpha : \omega \leq \alpha < \Omega\}$ is the class of denumerable ordinals. The union of these two is the <u>second (cumulative) number class</u>, denoted by "II". Note that II is the smallest class of ordinals that contains 0, is closed under the operation of taking successor, and is closed under the operation of taking the limit of an ω-type fundamental sequence.

If $\aleph_0, \aleph_1, \ldots, \aleph_\alpha, \ldots$ is the sequence of infinite cardinal numbers, arranged in order of magnitude, then $\omega_\alpha =_{df}$ the least ordinal whose cardinality is $\aleph_\alpha$.

<u>Recursive Function Theory</u>.

We shall adopt a Turing machine approach to recursive function theory, although this particular approach is not crucial to the subject matter presented. Every ver-