## Richmond Math Talk October 13, 2015

(\*) Greetings everybody! I'm so happy to be talking to you today, all of you here playing math. That's so awesome! And of course I'm with you, math should be played!

And actually I think of myself as something of an expert on this topic. As co-founder and head of product development at the ThinkFun game company, for the past 30 years my job specifically has been to play with math. That's what I do for a living.

So we have a program at ThinkFun where we study ways to help make math more fun and become easier to understand. It's called Brain Lab.

We're working now on a collection of next generation math games that are getting close to being launched. I thought it would be fun to describe them to you, and kind of give you a look behind the curtain at how we approach projects like this at ThinkFun.

So... For my talk today, I'm going to tell you a few stories about Math at Play, how we've done it so far at ThinkFun and how we're hoping to do it in the future.

(\*) Let me start with a little background. I'm with ThinkFun of course, my wife Andrea and I started ThinkFun 30 years ago, in 1985.

We make games that help build thinking skills while you play. (\*) Our original company name was Binary Arts, we changed to ThinkFun (\*) after nearly 20 years because everybody thought Binary Arts was a company that made software, not physical games.

- (\*) And we do make physical games! We make games like Rush Hour, (\*) like Laser Maze, (\*) like Gravity Maze. (\*) We also make MathDice, I'll talk more about this in a minute.
- (\*) Here is our mission statement...

"We translate the brilliant ideas of the craziest mathematicians, engineers and inventors into simple toys for boys and girls around the world."

We wrote that when we started, and that's still what we do today.

And we've had a lot of whacky inventors. (\*) Nob Yoshigahara was a brilliant Japanese guy, (\*) he invented Rush Hour. (\*) Here is our first inventor, Bill Keister. He had a lifelong dream to make a three legged pair of pants, finally he did it. (\*) He invented the Hexadecimal puzzle.

(\*) So I want to tell you about the MathDice inventor. MathDice was (\*) originally called Dice Mania, and it (\*) was invented by our son Sam... he invented this at an "Invent a Math Game Workshop" program when he was in 6<sup>th</sup> grade, at Burgundy Farms Country Day school in Alexandria. That was 16 years ago, by the way.

Sam came up with this game for a school assignment. I liked the game right away. We kept playing it at the breakfast table and just couldn't stop. Sam's younger brother Mike, a 3<sup>rd</sup> grader who was good at math... Mike learned exponents in about 15 minutes. To him, exponents was just another game rule he wanted to learn to get better.

Amazing, right? A lot of you already know MathDice, I know there was a workshop this morning so you may see it soon if you haven't already.

I'm not going to explain the game here... (\*) though I have to give just one MathDice example, I love this one. One time I was at a MathDice tournament, explaining the game to some hotshot father who wasn't impressed.

So I challenged him to a single roll, winner take all...

and I rolled... 2x4, Target 8. Scoring dice 2 4 and 5

Guy looks at me and says, How about 4+5-2 gets 7, that's one away.

Now there's a bunch of ways to get 7... but you can't get to 7 and a half, or any fraction... so he may have beaten me.

I look at the dice... look at him... something clicks.

(\*) 2^5/4 gives you 8, hits the target exactly. Boom.

So let me tell you the story of what happened to MathDice after that. For three years Sam and Mike and I kept playing at breakfast, just because it was fun

Then when Mike was in 6<sup>th</sup> grade, the school put me in charge of the Math Game Invention program that year. I ran a workshop where I used MathDice to demonstrate what a successful invention could look like... and all the kids loved it, they just wanted to keep playing.

- (\*) So we held a MathDice tournament between The Burgundy Farm 6<sup>th</sup> grade and the 6<sup>th</sup> grade class in Sam's new school at Maret, which was in D.C.. We bussed the Burgundy kids to Maret and set up tables on the front lawn, and had a big tournament!
- (\*) It was really crazy... (\*) the kids had so much fun playing.

- (\*) There is Mike playing in the Finals against the best player from the other school...
- (\*) Lots of tension, there were more than 50 kids all pressing in...
- (\*) And Mike won! His team carted him back to the bus on their shoulders, it was wild.
- (\*) The next year we held a bigger tournament, this time we invited eight Alexandria schools and held it at Barnes and Noble.

There's Sam... he's in 10<sup>th</sup> grade at this point. (\*) This tournament got crazy too.

- (\*) This is the championship round
- (\*) Here I am, announcing the winner...
- (\*) Here are our first Rubber Chicken Award Winners.
- (\*) Then Arlington county stepped in and said they wanted to run MathDice in their schools.

So for the past 12 years Arlington has been holding a county wide 5<sup>th</sup> grade MathDice competition, first Saturday morning in May each year... teams from all 22 Arlington schools train for months to get ready, it's incredible.

- (\*) Registration
- (\*) 22 Teams of kids play MathDice against each other

- (\*) Judges tallying scores
- (\*) Parent cheering sections
- (\*) And the musical skits the kids are doing now (\*) to get the Rubber Chicken Award!

## Amazing!

- (\*) And by the way... we did turn MathDice into a commercial game, it has been very successful for us and is still going strong.
- (\*) All this and more, because Sam invented Dice Mania 16 years ago!

So my next story is about (\*) how Visual Brainstorms came to be. Visual Brainstorms is a deck of 100 cartoon math cards...

(\*) Weird, in your face challenges like this one. Meant to be playful.

So this story actually starts (\*) with me as a kid, back when I was nine years old. The year was 1964... and over Christmas vacation somehow our family came up with this really funny idea to pretend that we had a secret identity as the Legion of Super Diseases. My brother Dennis, who was 23 and home from grad school, really got into this.

(\*) He drew cartoons and actually made an LOSD comic book.

So I asked him to help me to write my own LOSD stories... and he did! We would sit on the couch together, (\*) I would narrate my story and

He would type it, using a typewriter and onion skin paper... and then

(\*) He would use colored pencils and illustrate the stories!

(\*) My stories weren't particularly good... but his artwork was amazing, (\*) I really loved it, to find myself in (\*) a collaboration with somebody who was so talented.

So fast forward 30 years to 1995. ThinkFun is cranking along, bringing puzzles to market, and I'm out there looking for new ideas. Our chief product designer at the time could also draw cartoons really well... his style was kind of similar to my brother's actually.

So when an inventor came to us and said "I've got a fantastic collection of math challenges", I knew exactly what I wanted to do... (\*) which was to make an awesome collection of math puzzle cards with incredible fun cartoons!

- (\*) Here is an algebra word problem
- (\*) Here's a spatial imagination exercise
- (\*) I came up with this challenge myself... "It was nearly midnight, and the full moon was just rising..."

Does anybody know the answer? The full moon doesn't rise at midnight, it rises at dusk!

(\*) That's me, by the way. Captain Willy.

Now... each of these stories here started with a kid, your age or younger, who liked math and had fun playing around with ideas. Kind of makes you think.

All right... these next three stories I'll tell, are about this new math idea we are working on. We are trying to use games to do a better job of teaching core math concepts.

The big idea we have here... is that we want to develop game rules that are built directly into the fabric of a specific math system.

Then we need to make the games actually be fun, so you want to play a lot and get better. If we can do that... Then getting better at the game means that you are getting better at the math, naturally as part of the deal.

OK, so let's start with the math idea of... (\*) ALGEBRA! This is a big one, so I've got to start with a little background.

I've been taking a survey of my own friends – adult professionals – and almost nobody I've talked to (\*) has a confident answer even for what Algebra actually is.

There's a reason why this cartoon is funny...

There is definitely something weird about algebra. I spent a couple of years trying to sell ThinkFun games into schools, I ended up so frustrated (\*) that I had this cartoon custom made about my experience.

So people, let me tell you... I make games for a living. If my customers went out and published a cartoon like this, to show what they thought about one of our games, this would not be a good sign!

So let's get down to it.

- (\*) From the perspective of a game company... what is the essence of algebra?
- (\*) Well we know it involves equations, usually with numbers and variables like this.
- (\*) It can get really complicated,
- (\*) It often involves graphs though it doesn't have to...

Frankly, though... this kind of stuff reminds me of our expert level challenges in our multi-challenge games. And... what we tell our customers is... Please don't even try the expert levels until you learn to have fun playing with the beginners and the intermediates. You go in too fast and it will ruin the game for you.

(\*) So when you strip away all the complicated stuff... what Algebra is really all about, is the relationship between the left side of an equation and the right side of the equation. Ideally you want to bring the two sides into balance.

It can be symbols with an equal sign...

- (\*) And it can also be a balance scale.
- (\*) The ancient Greek mathematician Archimedes figured out the math behind the balance scale. It did this around 250BC.
- (\*) Force equals mass times distance... this is how a scale actually works. You lay down the weights, gravity kicks in and the forces offset and balance against each other across a fulcrum. This is algebra right here. Physical algebra.

So... that means if we develop a balance beam style puzzle, we will be developing a puzzle that is based on the underlying concepts of algebra itself. And that's just what we want to be doing.

- (\*) And so indeed, we are in the midst of developing our new Balance Beans game... a simple balance game for young kids, 5 and up.
- (\*) Here is a picture of the prototype. You place beans on the seesaw according to the rules on the instruction card, try to make it balance.

You can just play with your fingers and intuition... or, if you want (\*) you can solve it as an algebra problem and then place the beans where you have calculated they need to be.

These Balance Bean challenges are really fun to play, and on this size board they don't get too hard.

We could develop this into a fast paced online game and then hold tournaments where we set up leaderboards and offer prizes to the winners.

Imagine then... we do this, it starts to grow, then we start to introduce more advanced math into the rules.

- (\*) For example... how about if we changed to a logarithmic scale. How many of you know logarithms? This is just a fancy version of exponents! Just like Mike, you could learn this in 15 minutes if you had to, to play this game.
- (\*) We could do different shape weights. We could add a variable to change the density of the weights. We could customize the scale calibrations.

- (\*) If we're not careful we could end making it too complicated. This wouldn't be good.
- (\*) But if we choose good challenges, make it more like MathDice where you marvel at the math... then it becomes fun.

So that's my idea... Liberate algebra, show what it truly is, and have fun doing it.

For my next story... (\*) let's talk about mathematical proof!

This is another topic (\*) that makes adult professionals uneasy... hence this cartoon.

I googled "How to explain mathematical proof", and here is the first reference that came up. (\*) I screen captured the first paragraph... The first sentence reads...

"Performing mathematical proofs can be one of the hardest things for students to do."

Well... Isn't that playful? I can promise you, we wouldn't want this to be an Amazon review for any of our games... who wants that?

So let's stop this crazy talk. What is the essence of mathematical proof? Most fundamentally, it means that you have investigated the entire system and are able to account for every single case.

Proving it means that you own it. And that shouldn't be intimidating, that's empowering!

So a couple of years ago we did a pilot project with a couple of hundred students to see if we could teach them to do mathematical proofs and have fun with it.

- (\*) We introduced ThinkFun University,
- (\*) We made a cartoon video that described what we meant by a proof... that's me again by the way, on the right there...
- (\*) And we introduced our Chocolate Fix logical deduction puzzle.
- (\*) And invited kids to learn how to make their own Chocolate Fix puzzles that they could then
- (\*) Send to challenge their friends. Of course, it's a lot harder to create your own puzzles than it is to solve somebody else's puzzles. Again, it's a matter of ownership and empowerment.

There are a lot of issues, but one of the biggest is quality control... you have to make sure that you don't send out a puzzle that has more than one solution... or a puzzle that can't be solved at all. That wouldn't be fun. The way to do this is to perform a mathematical proof on your puzzle... that way you will know for certain.

(\*) So we developed a computer modeling program that can break down any Chocolate Fix challenge into a complete tree graph that illustrates each and every condition.

Here is a challenge that has a unique solution, there in green. That's good.

(\*) This challenge has two solutions. That's bad. You need to go back and repair this one.

(\*) This challenge doesn't have any solutions. That's bad too.

We taught players how these graphs worked then taught them to create their own graphs.

We did a deep dive into how to analyze these kinds of puzzles...

- (\*) We taught them the two kinds of decisions
- (\*) The three types of piece analysis
- (\*) The five contradiction types
- (\*) And when they got to the point where the kids could make their own puzzles, they loved it.

I'm pretty sure that these kids will do just fine with mathematical proof when they encounter it in school. At least it won't be a scary idea to begin with.

(\*) So this next story started just as a thought exercise, that came out of the Chocolate Fix project.

As we were playing around with simple online Chocolate Fix challenges, we were trying to figure out how to make lots of challenges which all have equal strength.

(\*) So one idea I came up with was that if we rotated the pieces around on various axes of symmetry, we could get puzzles that looked very different but in fact were the same thing.

Here you can see a sketch of how I took one Chocolate Fix challenge and rotated it around to create three new ones.

This led us to the question of just how many ways are there to actually do this?

- (\*) It turns out that there are eight different ways to transform a s square.
- (\*) Furthermore, it turns out that this is a well studied mathematical phenomenon, called "The symmetry group of the square". This is a Dihedral Group D-4. I had no idea!

When I learned about this it was a little bit like reading about those weird sea creatures that live a mile down in volcanic jets in the ocean... they are alive, but they live by very different rules. And you know, math is full of this kind of thing. This square symmetry system is just one example.

So the thing I liked best about this system, is that you can always get from any one face to any other face with only one transformation of your square. No kinks in this system, everything flows with perfect symmetry.

That means we can turn it into a game.

(\*) So we've been experimenting with what we're calling The Compositional Flip game.

This will be a "Follow the Leader" style game... (\*) you and your opponent – which is the computer - start with identical images. (\*) Your opponent then shows two moves it will make in succession... Turn 90 degrees, and Flip Vertical... and as fast as you can, you must signal

the one single move you must make to match orientation with your opponent. Flip Diagonal 2 will do the trick here.

It's intense... but it's a tight little game. And just like MathDice, you can build your mental muscles pretty fast. And once you do, you will be able to rotate objects in your head and understand the math that is being expressed as you do this.

- (\*) The crazy thing about this game is that it's actually algebra... it turns out there is a different type of algebra, called composition algebra... that works by adding motions together, rotations and reflections that normally you think about with geometry rather than algebra. So it's a hybrid idea.
- (\*) And it's got its own rules too. With this game, it matters which operation you execute on first. You get a different answer if you do them in reverse order.

This means that this algebra doesn't follow the commutative property. Huh!

(\*) So this new game idea, Compositional Flip... the game rules are actually the rules of how this Dihedral math system itself works. Like Chocolate Fix with mathematical proof and Balance Beans with algebra... that's what we want!

So that's what we've been working on. What we are hoping is that we can develop these into programs to help kids learn to construct their own mental models of what's actually going on, with algebra, with proof, with geometry, and with other forms of math too.

I want to end now with a couple of quick examples about how I use math to actually run our company.

The underlying point I want to make here... is that business and organization people tend to use math as a tool to help us make decisions, rather than using math for its own sake. So what becomes really important, is for you to get really good at all kinds of math so you are confident enough to know how to use it to get the best answers.

So one of the most important decisions we make at ThinkFun is to predict how many of each of our games we're going to sell by the end of Christmas each year. If you get this right, everything else tends to go smoothly. If you run out too soon, you lose revenues and have all kinds of headaches. If you buy too many, you tie up all your cash which is even worse.

So that's a biggie... and it's all about math.

Here are a couple of more simple examples...

This one is totally an algebra problem... (\*) We sell our games on Amazon.

There's two ways to do it... we can use our vendor account or our seller account. (\*) With the vendor account, Amazon buys our games outright and they sell them. That's the top one here... "Ships from and sold by Amazon". They buy games up front and Amazon does all the marketing.

With the Seller account — "Sold by ThinkFun and fulfilled by Amazon" - we make more revenues but then pay fees to Amazon, and we have to do our own promotion.

Which is the better choice, vendor or seller account? This is a math question. It's an algebra question.

And speaking of Amazon... one more example, different style of math.

(\*) One of our new games this year is Compose Yourself... it's this crazy game that lets you compose your own music. You buy the deck of cards at Amazon and then come to our website for an amazing online experience.

We took a big gamble with this game... we are actually selling it exclusively through our Seller account on Amazon... no place else, you have to go there.

Whether this strategy works or not is its own story... the reason it's a math story is that we have done something very new here, (\*) we have set it up so we can measure the entire digital footprint of how customers are purchasing the game and how users are coming back to play the online app after they have purchased. We control everything and can get instantaneous full information.

So it turns out that our son Mike – the MathDice champion - now works at ThinkFun, he's leading our digital strategy team. He's watching these graphs closely, this is a whole new way of doing analytics. To me it feels like these are spider webs, and we're going to be able to register every squiggle and react way faster than ever before. This is math!

Now the numbers here are tiny... but this is the kind of thing they are thinking of when people talk about Big Data... tons of information that you need to organize into patterns that you can use. And just to complete the loop here... Sam the MathDice inventor is now 28, he's become a coder and he works on Big Data.

And speaking of Sam, that brings me to my last math story. This is my favorite one, I hope it's as fun for you as it is for me.

(\*) This one goes back to Sam back in 6<sup>th</sup> grade again. That's Sam there, manning one of the games at the Burgundy Farms Fall Fair. So the year before this, I joined the Games Committee for the Fair. I did my volunteer time... then the woman in charge told me that she wanted me to take over and run it the following year.

When I started, the games were really lame. To give you an idea... the biggest money maker was a piece of firewood that somebody had drilled a bunch of holes into and then plunked lollipops into the holes. You could win a prize, but basically the game was that you paid 50 cents to get a lollipop.

As soon as I could I decided to make things more playful. A bunch of parent volunteers were assigned to me, and also all the kids from Sam's 6<sup>th</sup> grade... there was going to be a lot of work, collecting tickets and managing all the games and everything.

I'll always remember the advice that one of the parents who had been in charge before, whispered to me. "Watch out, you can't trust the kids to run these games", she said. That struck me as weird, I knew Sam and a lot of the kids in the class, and they seemed pretty responsible to me.

- (\*) So first I went to work on the games. We replaced the firewood lollipop game with big foam sculptures of a strawberry and a couple of mushrooms that the kids decorated in art class. It's the same game as the log game... but we had removable lids for easy refilling, we were all over it. Suddenly it was fun, we got twice as much business as the old version had done.
- (\*) We dressed up the old baseball throw game...

- (\*) Somehow we got a hold of a toilet and made a game where you threw toilet paper into it.
- (\*) Everybody's favorite game, though... was the Barney Launch, a catapult where you shot stuffed Barneys into a volcano.
- (\*) Here's the Prize Booth, that was a big part of the event.

The week before the Fair, I got together with the class and we talked about why we were doing the Fair at all. I had learned from the head of school that there were two purposes: first was community relations, be a good neighbor. The second goal was to raise money for the student scholarship fund. We talked about how these goals were different, and how we needed to understand that we had to achieve both goals together.

The first year was a big success, the kids turned out to be very responsible. I got parents to count tickets at the end of the day, we collected a lot of data that way.

A few weeks after the Fair, I came in and ran a math workshop with the kids. I showed them the data we had collected, organized into a bunch of spreadsheets.

- (\*) I showed the tickets sold and plays per game.
- (\*) I showed performance against goal
- (\*) We did revenue and income reports
- (\*) Tracking our prizes, a lot went into that.

We went beyond this too. As a group, we went through and simulated time and motion studies for each game... some games play fast like picking a lollipop, and some games like Barney Launch play slow. We calculated things we called "satisfaction measures" based on how much total time was spent on a game by everybody over the whole day.

(\*) Then we talked about our larger goals, and asked ourselves what we needed to improve on or to change in order to meet these goals. It was really fun... I felt like the kids really got it, they actually experienced how math could be used to help understand and achieve larger objectives.

So I was in charge for three years... my last year was when Mike got to 6<sup>th</sup> grade. I'll always remember, during our first meeting to get started that year, a kid raised his hand. I called on him and he said, "You know, you can't trust the parents to run these games..."

Now that, to me... is the power of Math at Play.

Thank you.